Physics course:
General Wave Physics

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1D Propagation (1/2)

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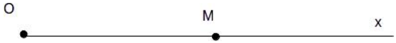

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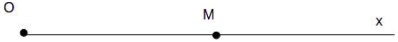

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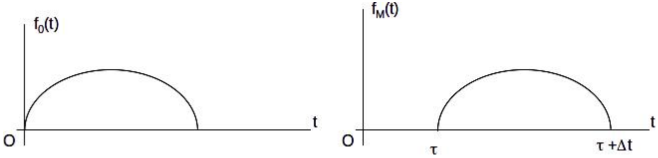

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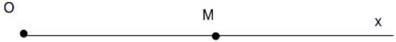

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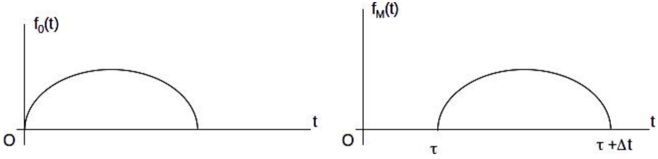

Definition

The medium is assumed to be **non-dispersive** when the delay τ of M with respect to O is proportional to the distance $OM = x$, and therefore we can write $\tau = \frac{x}{c}$.

The parameter c has the dimension of a velocity: is the propagation speed of the wave.

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Non-distorted and non-attenuated propagation from O to M is written:

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Remark

If F takes the same value for two spatial positions, we may assume:

$$t_1 - \frac{x_1}{c} = t_2 - \frac{x_2}{c}$$

if $t_2 > t_1$, then $(x_2 - x_1) = c(t_2 - t_1) > 0$

PPHW: Definitions (1/2)

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Physical properties of a wave are described by a mathematical function called the **wave function**:

$$\Psi(x, y, z, t)$$

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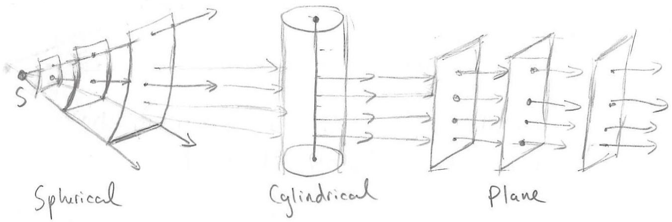

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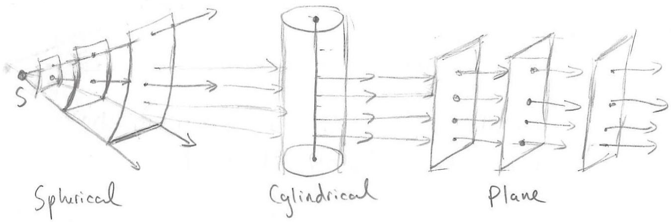

Plane wave approximation

At far distances straight rays are quasi-parallel. If the propagation is unique at any point of space for all time, then the value of the quantity displacing depends on time but does not depend on the point considered in any plane (P) orthogonal to the propagation direction. Such plane (P) is called a **wave plane** and such wave is called **Plane Wave**.

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- $\Phi = \omega\left(t - \frac{x}{c}\right) + \varphi$: phase of wave at fixed position and time.

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Summary

$$\Psi(x,t) = A\cos\left[\omega t - kx + \varphi\right]$$

with:

$$k = \frac{\omega}{c}$$

and

phase velocity $c = \frac{\omega}{k} = \frac{2\pi f}{2\pi\sigma} = \lambda f = \frac{\lambda}{T}$ (m.s^{-1})

Wave Equation

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$$\frac{\partial^2 \Psi}{\partial x^2} = \frac{1}{c^2} \frac{\partial^2 \Psi}{\partial t^2}$$