Physics course:
Mechanical Waves

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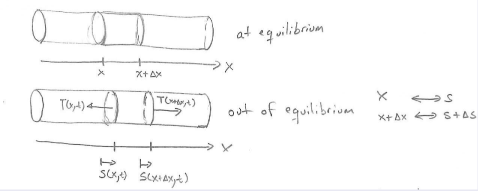

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- c: velocity (m.s^{-1})
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For aluminum, $E = 70$ GPa, $\rho = 2700$ $kg.m^{-3}$ and $c \approx 5092$ $m.s^{-1}$

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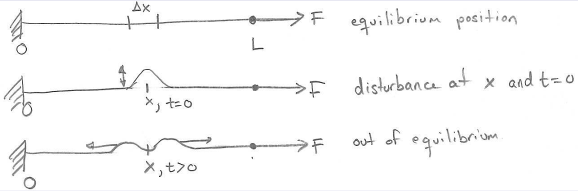

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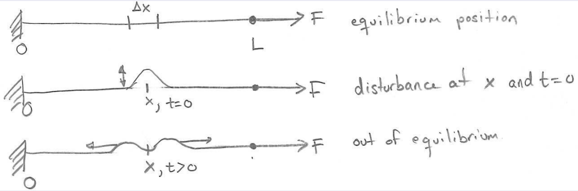

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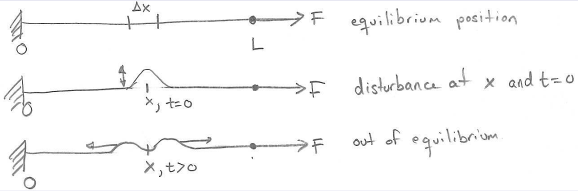

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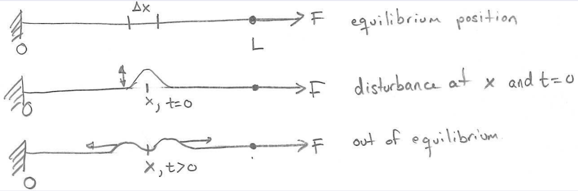

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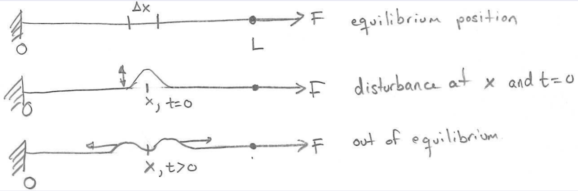

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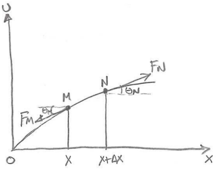

- Its weight, $\Delta m \cdot \vec{g} = \mu \cdot \Delta x \cdot \vec{g}$ (neglected)
- The tangential force F_M exerted in M by the left side of the string, $\vec{F_M} = -\vec{T}(x,t)$
- The tangential force F_N exerted in N by the right side of the string, $\vec{F_N} = \vec{T}(x+\Delta x,t)$

$\vec{T}(x,t)$: the tension of the string from Newton's third law.

$|\vec{T}(x,t)| = T(x,t)$

$\theta_M = \theta(x,t)$

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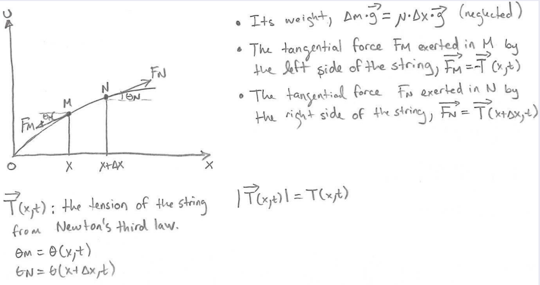

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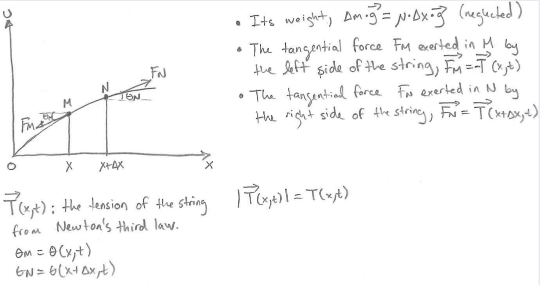

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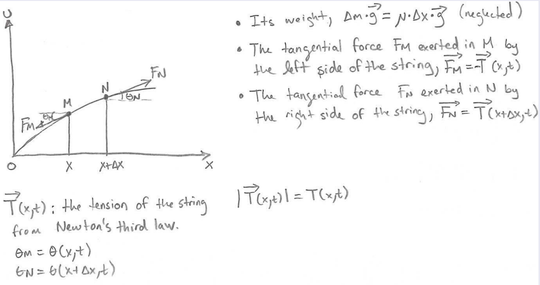

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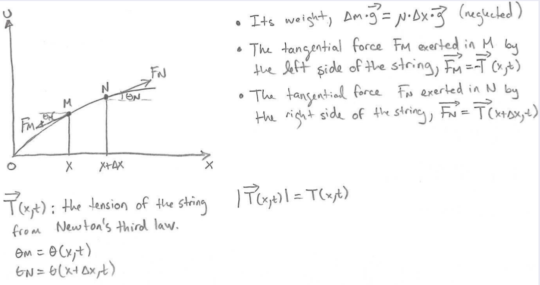

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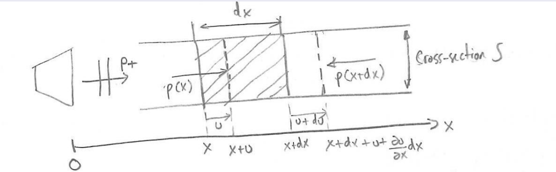

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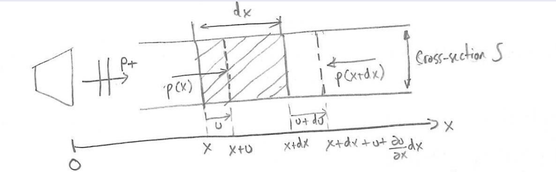

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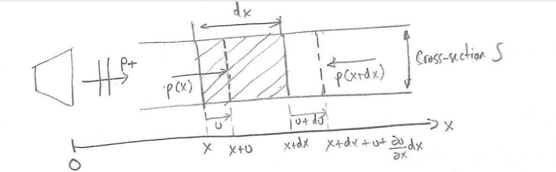
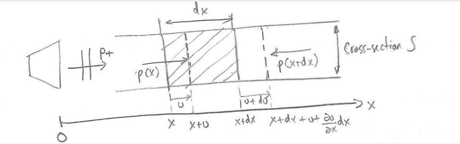


- We consider sound propagation through a pipe of cross-section S containing a perfect fluid.
- The vibration is induced by a piston shaking at $x = 0$, which transfers its movement to the fluid slices close to it.

Propagation of Sound Waves in Fluids (2/3)

Description of the propagation mechanism



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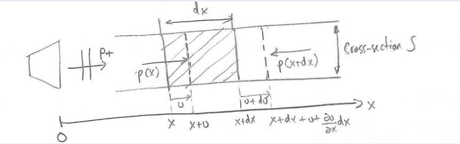

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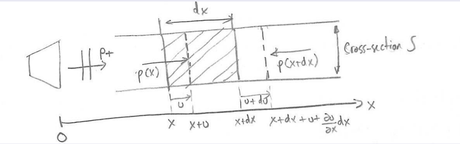

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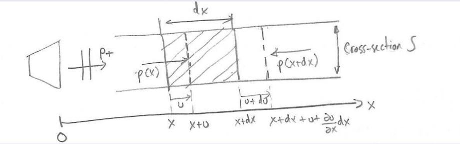

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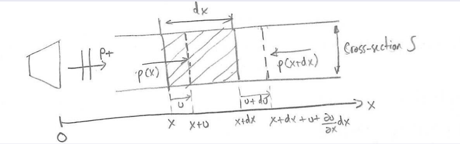

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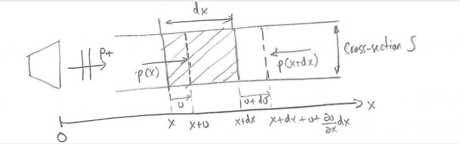

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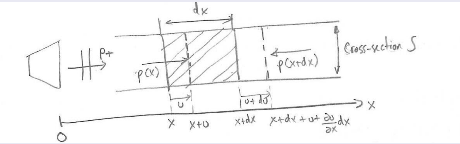

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During dt, the slice $x + dx$ will displace at first approximation $u + du$ and so, $u(x + dx, t) = u(x, t) + \frac{\partial u}{\partial x} dx$.

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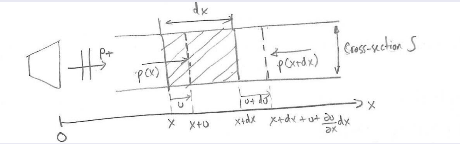

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As a result, during infinitesimal time dt, the fluid slice at x will displace a quantity $u(x,t)$ (*enlongation* of movement).

During dt, the slice $x + dx$ will displace at first approximation $u + du$ and so, $u(x+dx,t) = u(x,t) + \frac{\partial u}{\partial x} dx$.

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Propagation of Sound Waves in Fluids (2/3)

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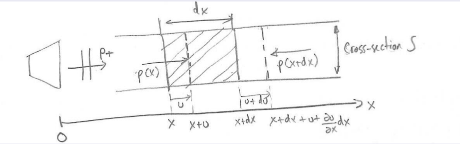

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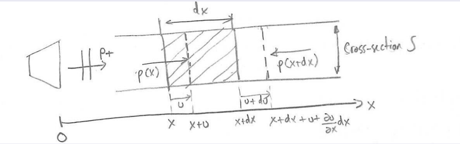

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